SCATTERING FROM A LARGE HOLE OF ANY SHAPE IN A MULTIMODE WAVEGUIDE

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Abstract

An upper bound for the reflection coefficient of any mode incident on a hole of arbitrary shape in the infinitesimally thin wall of a perfectly conducting multimode waveguide is given. The theoretical value is verified experimentally.

Introduction

The scattering of a waveguide mode from a small hole in the infinitesimally thin wall of a perfectly conducting multimode waveguide has been calculated by many authors using perturbation theory[1.11] This perturbation theory is valid for holes whose cross sectional area is much less than λ^2 where λ is a free space wavelength. In order to compute the scattering of a waveguide mode from a hole whose cross-sectional area is larger than λ^2 one must solve a difficult boundary value problem. In this paper an expression is derived for the upper bound to the reflection coefficient of any mode incident on a hole of arbitrary size and shape in the infinitesimally thin wall of a perfectly conducting multimode waveguide. The expression for the upper bound is independent of the complex amplitude of the equivalent distribution of magnetic surface current in the aperture of the hole and depends only on the relative amplitude distribution of this current. Therefore, the upper bound can be computed if the relative distribution of equivalent magnetic surface current in the a aperture of the hole can be estimated. In this paper the upper bound is computed for the special case of TE_{01} mode incident on a resonant circularly symmetric circumferential gap in the infinitesimally thin wall of a perfectly conducting circular waveguide. Using the solution to the boundary value problem of a slot antenna in a perfectly conducting infinitesimally thin plane, the equivalent distribution of magnetic surface current is estimated to be a sine function of axial position that vanishes at the ends of the gap. The resonant wavelength is assumed to be equal to the period of the sine function. The calculated result compared very well with the value measured using a pulsed reflectometer test set. This work was helpful in evaluating a fault location scheme for a millimeter waveguide transmission system under development.

Theory

The upper bound of the reflection coefficient R for the TX mode is computed as follows: An expression for R is obtained using the conservation of power principle applied to a hole in a perfectly conducting circular waveguide with an infinitesimally thin wall excited by a single TX mode. Let P_{TX} be the power in the incident TX mode, P(reflected) be the total power that is reflected from the hole, P(transmitted) be the total power that is transmitted past the hole, and P(radiated) be the total power that is radiated through the hole. Then the conservation of power can be expressed as

$$P_{TX} = P(\text{reflected}) + P(\text{transmitted}) + P(\text{radiated}). \quad (1)$$

Let P^-_{TX} be the power reflected in the TX mode, P_2 be the power that is transmitted in the TX mode past the hole, and 1+t be the complex voltage transmission coefficient of the TX mode. The following equations are then valid

$$R = \frac{P^-_{TX}}{P_{TX}} \quad (2)$$

and

$$P_2 = |1+t|^2 P_{TX} \quad (3)$$

where || denotes the magnitude. Let P_1 be the power transmitted in modes other than the TX mode. Substituting Eq. (3) into Eq. (1) we obtain

$$P_{TX} = P(\text{reflected}) + P(\text{radiated}) + P_1 + |t|^2 P_{TX} + 2\text{Re}[t]P_{TX} + P_{TX} \quad (4)$$

where Re denotes the real part. The term $|t|^2 P_{TX}$ is the power scattered in the TX mode in the forward direction, P',

$$P' = |t|^2 P_{TX}\ . \quad (5)$$

The sum of the first four terms of Eq. (4) is the total power scattered by the hole, P,

$$P = P(\text{reflected}) + P(\text{radiated}) + P_1 + P'. \quad (6)$$

Let ϕ be the phase of the forward scattered TX mode with respect to the incident TX mode. Using Eq. (6) and Eq. (4) we have

$$|t| \cos \phi = - \frac{P}{2P_{TX}} . \tag{7}$$

Using Eqs. (7), (5), and (2) we obtain

$$R = \frac{4 \cos^2 \phi P' \bar{P}_{TX}}{P^2} . \tag{8}$$

The upper bound is given by Eq. (8) with $\phi = \pi$,

$$R < \frac{4P' \bar{P}_{TX}}{P^2} . \tag{9}$$

Equations (8) and (9) are valid for holes of any size and shape and are dependent only on the relative amplitude of the equivalent distribution of magnetic surface current. Equations for P' and $\bar{P}_{TX}$ can be obtained in terms of $\bar{K}_{eq.}$, the equivalent distribution of magnetic surface current, where

$$\bar{K}_{eq.} = \bar{n} \times \bar{E} . \tag{10}$$

$\bar{n}$ is an outward unit normal to the waveguide wall and $\bar{E}$ is the electric field in the aperture of the hole. P is given as the sum of the power radiated through the hole and the power carried by all modes excited by $\bar{K}_{eq.}$

The upper bound was calculated for a pure TE_{01} mode incident on a circularly symmetric circumferential gap in a circular waveguide. Let the axis of the waveguide lie on the z axis of a cartesian coordinate system, A be a complex amplitude, k be the propagation constant of free space and n be an integer. It is assumed that $\bar{K}_{eq.}$ is given by

$$\bar{K}_{eq.} = \begin{cases} \bar{i}_z \; A \sin (kz) & |z| \leq \frac{n\lambda}{2} \\ 0 & |z| > \frac{n\lambda}{2} \end{cases} . \tag{11}$$

The total power that is scattered inside the waveguide is approximately equal to the total power that is radiated through the hole if the waveguide circumference is many free space wavelengths long; this result was confirmed numerically. Let a be the radius of the waveguide, J_1 be a Bessel function of the first kind of order one, $u_{TE_{01}}$ be the first nonzero root of J_1, $h_{TE_{01}}$ be the propagation constant of the incident TE_{01} mode in the z direction. Then the bound on R is given by

$$R \leq \frac{\frac{4}{\pi^2} \left[\frac{k^2 a}{h_{TE_{01}} \, n u^2_{TE_{01}}} \right]^2 \sin^4 \left\{ \frac{n\pi}{2} \left[1 + \frac{h_{TE_{01}}}{k} \right] \right\}}{J_1^2(n\pi)} . \tag{12}$$

The value of Eq. (12) in decibels is plotted versus n in Fig. 2 at a wavelength of 5 mm.

Experiment

The theory presented was verified experimentally using a pulse-reflectometer shown as a block diagram in Fig. 3.[12] The klystron oscillator is modulated with a pulse whose duration is typically 15 nanoseconds. The pulse passes through the circulator, tuner, transducer, taper, and then into the circular waveguide of 50.8 mm inside diameter. The reflected pulse passes back through the taper, transducer, tuner and circulator and reaches the detector through the precision attenuator and wavemeter. The reflected pulse is measured with respect to the reflected pulse produced from a short circuit. The system sensitivity using this setup is about -55 db. The experimental data was obtained by R. G. Fellers and is plotted in Fig. 2. The agreement between the theoretical and experimental results is good.

References

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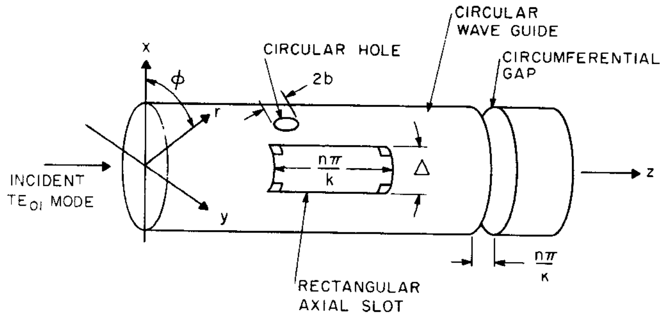


FIG 1

Geometry of Waveguide with Holes

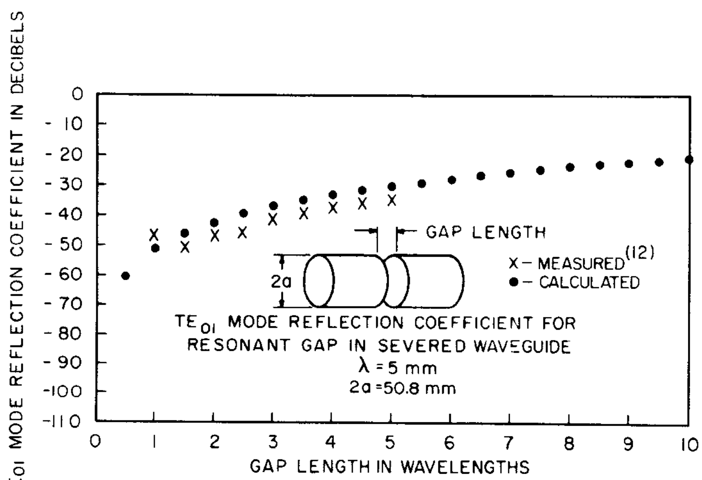
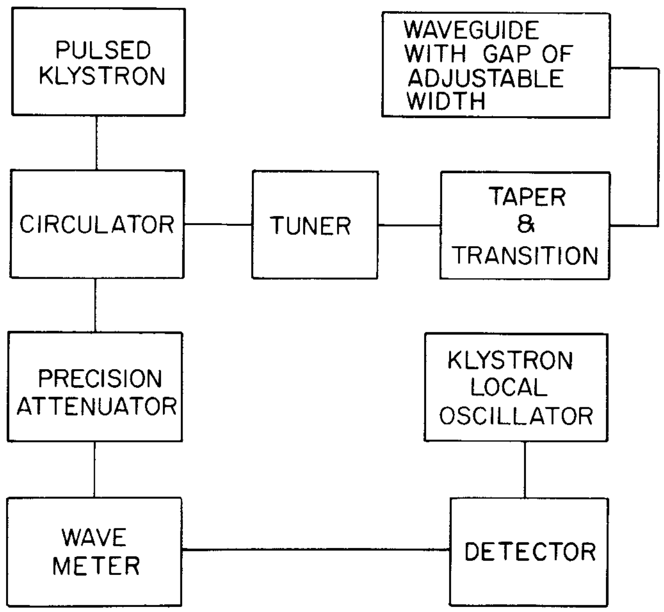


FIG. 2

TE_{01} Reflection Coefficient versus Gap Length



PULSED REFLECTOMETER SYSTEM
FIG. (3)